9th International Colloquium on Bluff Body Aerodynamics and Applications
University of Birmingham, UK
20 - 23 July 2020

Vortex induced vibration of a circular cylinder through its slow phase dynamics: theory vs. experiments

Francois Rigo[1,2], Thomas Andrianne[1], Vincent Denoël[3]

[1]*Wind Tunnel Lab, University of Liège, Liège, Belgium, francois.rigo@uliege.be*
[2]*FRS-FNRS, National Fund for Scientific Research, Liège, Belgium*
[3]*Structural & Stochastic Dynamics, University of Liège, Liège, Belgium*

SUMMARY:
The wake-oscillator model of Tamura for Vortex induced vibration (VIV) under free vibration conditions is deeply analysed for parameters identification through a perturbation methods analysis. In the slow phase model of VIV, a simple analytic solution for the amplitude of motion and phase is derived. The model is proved to be accurate for the estimation of the maximum amplitude and lock-in range. Results show that the phase between the displacement and the wake variable plays a key role in the lock-in mechanism and needs to be measured experimentally in further studies. The proposed parameter identification is direct thanks to analytical expressions and will be used in further experiments involving other shapes and tandem cylinders.

Keywords: Vortex induced vibration, perturbation methods, wind tunnel experimental testing

1. INTRODUCTION

The two dimensional model of Tamura (Tamura, 1981) has been widely used and has the advantage of a direct physical interpretation of each term. This set of two equations relies on Birkhoff's concept about the oscillating wake. The main outputs of the VIV study are (i) the maximum displacement amplitude and (ii) the size of the lock-in region to predict entirely the instability. Instead of solving numerically the system, the aim of this study is the derivation of an analytical solution which is useful to obtain directly these two outputs and understand driving mechanisms of VIV.

2. METHODS

2.1. Mathematical modelling

The wake-oscillator model for a circular cylinder is expressed as (Tamura, 1981):

$$Y'' + \left(2\xi + m_r(f + C_D)\frac{\Omega}{2\pi St}\right)Y' + Y = -\frac{f m_r \Omega^2}{(2\pi St)^2}\alpha \tag{1}$$

$$\alpha'' + 2\beta\Omega\left(\frac{4f^2}{C_{L0}^2}\alpha^2 - 1\right)\alpha' + \Omega^2\alpha = -\lambda Y'' - \Omega 2\pi St Y' \tag{2}$$

The meaning of all symbols is given in Appendix. Instead of solving this system numerically, a perturbation method is used. This requires the identification of *small* numbers in the physical quantities. Experimentally, the dimensionless transverse amplitude Y is limited to *small* oscillations, typically 0.1-0.3 (2S vortex shedding regime (Williamson and Roshko, 1988)). This motivates to express $Y = \varepsilon\tilde{Y}$, where $\varepsilon \ll 1$ and $\tilde{Y} \sim 1$. The mechanical damping is also small in structures submitted to VIV (typically in the range $0.1-1\%$), it is then expressed as $\xi = \varepsilon\xi_0$. The fluid to solid ratio for light steel structures submitted to air is typically very small ($m_r \sim 10^{-4}-10^{-3}$) and can be expressed as $m_r = \varepsilon^2 m_{r,0}$. The bifurcation parameter of the system is the airspeed, expressed as a ratio of speeds or frequencies with Ω. The system is solved to obtain the VIV response, thus in the lock-in range and close to the critical speed $\Omega \sim 1$ and it can be expressed as a small mistuning around 1: $\Omega = 1+\xi\delta$ because ξ is a small parameter, δ being the mistuning parameter of order 1. Assuming a small coupling, terms in the righthand side are of the order of ε. The wake coordinate α is expressed as $\tilde{\alpha} = \alpha/\alpha^*$ with $\alpha^* = C_{L0}/2f$ to simplify the nonlinear term in Eq. (2). The total damping ξ_t is the sum of the mechanical (ξ) and aerodynamic damping ($\xi_a = m_r(f/C_D)\Omega/4\pi St$). Noting that Eq. (2) is a Van der Pol type equation, the small parameter ε multiplies the non linear term and ε is chosen as $\varepsilon = 2\beta$. The short and compact version of the governing equations is:

$$\tilde{Y}'' + 2\xi_t\tilde{Y}' + \tilde{Y} = 2\varepsilon M_0\tilde{\alpha} \tag{3}$$

$$\tilde{\alpha}'' + \varepsilon\Omega\left(\tilde{\alpha}^2 - 1\right)\alpha' + \Omega^2\tilde{\alpha} = 2\varepsilon(A_0\tilde{Y}'' + A_1\Omega\tilde{Y}') \tag{4}$$

where $M_0 = -\frac{m_r C_{L0}}{8\pi^3\varepsilon^2}\left(\frac{\Omega}{St}\right)^2 = -\frac{C_{L0}\xi\Omega^2\pi}{2\varepsilon^2 SG}$, $A_0 = -\frac{\lambda f}{C_{L0}}$ and $A_1 = -\frac{2\pi St f}{C_{L0}}$. These equations are now expressed using two variables of order 1 ($\tilde{Y}$ and $\tilde{\alpha}$) and physical/experimental coefficients (A_0, A_1 and M_0). By using a multiple scale approach (averaging), a solution is sought with two time scales $t_1 = \tau$ (fast) and $t_2 = T = \varepsilon\tau$ (slow). An Ansatz is used for both variables: $\tilde{Y} = \tilde{Y}_0 + \varepsilon\tilde{Y}_1 + \mathscr{O}(\varepsilon^2)$ and $\tilde{\alpha} = \tilde{\alpha}_0 + \varepsilon\tilde{\alpha}_1 + \mathscr{O}(\varepsilon^2)$ and injecting it into Eq. (3)-(4) gives, at leading order:

$$\tilde{Y}_0 = R_y(T)\cos(\tau + \phi(T)) \tag{5}$$

$$\tilde{\alpha}_0 = R_\alpha(T)\cos(\tau + \phi(T) + \psi(T)) \tag{6}$$

For both degrees of freedom, the solution at leading order is a fast oscillation modulated by a slow envelope. Solving the system for the envelopes R_y, R_α and the phase ψ, secularity conditions read:

$$R_y' = M_0 R_\alpha \sin\psi - \xi_0 R_y \tag{7}$$

$$R_\alpha' = A_0 R_y \sin\psi + A_1 R_y \cos\psi - \frac{R_y^3}{8} + \frac{R_y}{2} \tag{8}$$

$$\psi' = \left(A_0\frac{R_y}{R_\alpha} + M_0\frac{R_\alpha}{R_y}\right)\cos\psi - A_1\frac{y}{R_\alpha}\sin\psi + \xi_0\delta \tag{9}$$

In steady-state condition, all lefthand sides vanish and solving for ψ gives:

$$\cot^3\psi + \delta\cot^2\psi + (1+D_0)\cot\psi + \delta - D_1 = 0 \tag{10}$$

$$R_y = 2\frac{M_0}{\xi_0}\sin\psi\sqrt{1 + 2\xi_0 D_0\sin^2\psi + \xi_0 D_1\sin 2\psi} \tag{11}$$

$$R_\alpha = 2\sqrt{1 + 2\xi_0 D_0\sin^2\psi + \xi_0 D_1\sin 2\psi} \tag{12}$$

where $D_0 = \frac{A_0 M_0}{\xi_0^2} = \frac{\pi\lambda f\Omega^2}{2\xi SG}$ and $D_1 = \frac{A_1 M_0}{\xi_0^2} = \frac{\pi^2 St f\Omega^2}{\xi SG}$ (always positive). Knowing physical parameters and constants in D_0 and D_1, the phase can be directly computed as a function of the mistuning δ by solving the third order algebraic Eq. (10). Then, the envelope of the displacement is deduced from Eq. (11) and presented in section 3.

2.2. Wind tunnel set-up

The wind tunnel set-up consists in a circular cylinder suspended horizontally and free to oscillate vertically. This aluminium cylinder has an external diameter of $D = 5$ cm, a thickness of 1.5 mm and a length of 1440 mm. It is supported by extension springs connected to a rigid frame attached to the ceiling of the test section of the Wind Tunnel Laboratory of University of Liège. The stiffness in flexion in the vertical direction is equal to 6155 N/m, elastomers are added and a wind-off analysis showed a natural frequency of $f_0 = 9.59$ Hz and a damping ratio of $\xi = 0.04\%$. The set-up is instrumented with one accelerometer measuring the vertical acceleration (acquisition frequency is set to 201.03 Hz). A cobra probe is installed in the wake of the model to measure the flow velocity (acquisition frequency of 250 Hz). A vane anemometer measures the wind velocity seen by the model. The Reynolds number is in the range $Re = 3 \cdot 10^3 - 3 \cdot 10^4$. Moreover, taps around the cylinder measure unsteady pressure signal at 250 Hz, synchronized with the accelerometer, to obtain the phase between the displacement and the lift.

3. RESULTS

The perturbation method presented in section 2.1 showed the key role of the phase ψ in the results. Eq. (10) is solved in Fig. 1 to obtain the cotangent of the phase $\cot\psi$ as a function of the mistuning δ for different values of the constants D_0 and D_1. The case $D_1 = 0$ is shown to make the link with other kind of VIV models that use only one coupling term in the fluid equation (only the acceleration (Denoël, 2020; Facchinetti et al., 2004) or the velocity (Hartlen and Currie, 1970)). When $\cot\psi = 0$, the phase $\psi = \pi/2$, leading to a high energy transfer and resulting in transverse vibration (critical speed when $\delta = 0$). For $\cot\psi$ *close* to 0, the phase is still close to $\pi/2$ and the structure vibrates as well, in an interval of airspeed called the lock-in range. It is observed from Fig. 1 that $D_1 = 0$ (black lines) gives a symmetric lock-in. The critical value of D_0 is 8 (from Cardano formula): there is an hysteresis (three real roots in Eq. (10)) in $\cot\psi$ for $D_0 > 8$ meaning that: (i) two stables branches are present depending of an increasing or decreasing δ and (ii) one unstable branch is present between the two others. The effect of D_1 is an asymetrisation of the lock-in range with respect to $\delta = 0$, that grows with D_1. For a sufficiently high D_1, the hysteresis for $\delta < 0$ created by $D_0 > 8$ can disappear.

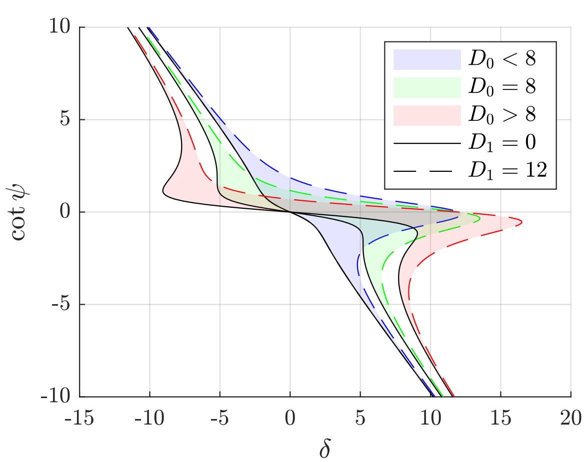

Figure 1. Phase vs mistuning: effect of D_0 and D_1 on the solution of Eq. (10)

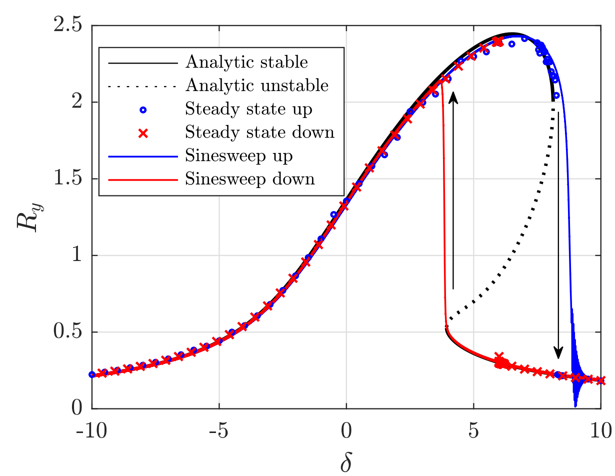

Figure 2. Amplitude vs mistuning: comparison of analytic (black) and numerical (red, blue) results ($D_0 = 1$, $D_1 = 8$)

Figure 2 shows the envelope of the transverse displacement Ry as a function of the mistuning δ for $D_0 = 1$ and $D_1 = 8$. These values where chosen to have only one hysteresis in the right part of the VIV curve and a case study for comparison with simulations. Black lines are analytical results obtained by substituting $\cot\psi$ in Eq. (11). The blue and red lines are the transient numerical solutions of the full model by slowly ramping up or down the mistuning. The blue dots and red crosses are numerical results from the full model (Eq. (1)-(2) or (3)-(4)) representing the steady state solution, using initial conditions chosen as steady state solution of the previous one. The bifurcation parameter δ is ramped up or down.

4. CONCLUSIONS

An analytical solution of the wake-oscillator model has been derived thanks to a perturbation method analysis. The value of f (in D_0 and D_1) can be estimated thanks to the fitting of the model on experiments, presented in the full paper. The importance of the phase in the VIV behaviour has been stated and its identification will be carried out in further experiments, from separation points identification and from phase measurement between displacement and lift. The parameter identification will allow to extend the study to other shapes and tandem cylinders.

ACKNOWLEDGEMENTS

This study was made possible thanks to the support of the National Fund for Scientific Research of Belgium (FNRS).

REFERENCES

Denoël, V., 2020. Stochastic analysis of vortex induced vibrations by means of a randomised wake-oscillator model. Proceedings of Second International Symposium on Flutter and its Application, ISFA2020.

Facchinetti, M., Langre, E. de, and Biolley, F., Feb. 2004. Coupling of structure and wake oscillators in vortex-induced vibrations. Journal of Fluids and Structures 19, 123–140.

Hartlen, R. T. and Currie, I. G., 1970. Lift-oscillator model of vortex-induced vibration. Journal of the Engineering Mechanics Division 96, 577–591.

Tamura, Y., 1981. Wake-Oscillator Model of Vortex-Induced Oscillation of Circular Cylinder. Journal of Wind Engineering 1981, 13–24.

Williamson, C. and Roshko, A., 1988. Vortex formation in the wake of an oscillating cylinder. Journal of Fluids and Structures 2, 355 –381.

NOMENCLATURE

C_D	drag force coefficient	$Sc = \pi^2\xi/m_r$	Scruton number
C_{L0}	amplitude of oscillation of the lift coefficient	$SG = 4\pi^2 St^2 Sc$	Skop-Griffin number
$D_0 = \pi\lambda f\Omega^2/2\xi SG$	parameters related to physical characteristics	$St = f_0 D/U$	Strouhal number
$D_1 = \pi^2 St f\Omega^2/\xi SG$	and experimentally identified coefficients	U	airspeed (m/s)
f	lift coefficient per unit rotation of the	$U_r = U/f_0 D$	reduced airspeed
	equivalent wake coordinate (Magnus effect)	$U_{cr} = f_0 D/St$	critical VIV speed (m/s)
f_0	natural frequency of transverse vibration (Hz)	y	transverse displacement of the oscillating cylinder (m)
f_{vs}	vortex shedding frequency (Hz)	$Y = y/D$	dimensionless transverse displacement of the cylinder
D	cylinder diameter (m)	α	inclination of the wake (rad)
L^*	dimensionless half length of wake-oscillator	$\beta = f/2\sqrt{2}\pi^2 L^*$	damping ratio of wake oscillator
m	mass per unit length of the oscillating	$\lambda = 1/(0.5+L^*)$	constant
	body (kg/m)	μ	air dynamic viscosity (kg/m/s)
$m_r = \rho\pi D^2/4m$	fluid to solid mass ratio	ρ	air density (kg/m^3)
R_y, R_α, ψ	slow dynamics state variables (displacement,	Ω	ratio of U to U_{cr} (or f_{vs} to f_0)
	wake and phase)	ξ	mechanical damping ratio
$Re = \rho UD/\mu$	Reynolds number	$(\cdot)' = d(\cdot)/d\tau$	derivative with respect to dimensionless time $\tau = 2\pi f_0 t$